# Pooled t-Test Procedure

## Pooled t-Test

The pooled t-test is a statistically based procedure to evaluate the variability in the mean (average) of test results between 2 sets of data. When QMP, Aggregate for Concrete Pavement is specified and the contractor chooses option 1, in the special provision, the contractor's test results tabulated from the sieve analysis for gradation may be evaluated and compared to the engineer's test results of the relocated aggregate, if the aggregate is relocated. This procedure will only apply to those contracts where the aggregate is produced at one location then moved to a new location. This procedure is a tool that may be used to compare the test results mean of the original stockpile (contractor's data) to the test results mean of the relocated stockpile (engineer's data). A failed comparison between the original aggregate and the relocated aggregate may be the result of segregation, contamination, or degradation that occurred in the relocation/re-stockpiling process. The engineer will make the final determination on the quality of the material.

On the following pages a step-by-step procedure illustrates how to compute the F statistic, which you will then compare to the CRITICAL VALUES FOR F DISTRIBUTION (F critical table provided). If the F critical is greater than the F statistic you computed you are sure with a 99 percent confidence level that the relocated stockpile is the same as the original stockpile.

NOTE: The minimum number of tests required on the relocated stockpile is 5 tests or 20 percent, whichever is greater, of the tests taken on the original stockpile. Therefore, if 77 tests are taken on the original stockpile you need to take at least 15 tests on the relocated stockpile.

A sample calculation of the F statistic is provided and a comparison is made to the F critical. In the example provided, the pooled t-test confirms that the stockpiles are the same.

# Pooled t-Test Procedure
# (One-Way Analysis of Variance)

## 1.) Calculate Average Test Results ( A ) for each stockpile:

$A1 = \Sigma\sigma 1 / n1$ $\quad\quad$ $A2 = \Sigma\sigma 2 / n2$

$\sigma$ : the individual test result

n : the number of tests performed on that stockpile

1 : original stockpile

2 : moved stockpile

## 2.) Calculate the Grand Mean (T ) for Pooled Data:

$T = (\Sigma\sigma 1 + \Sigma\sigma 2) / (n1 + n2)$

## 3.) Calculate the Treatments Sum of Squares (SST):

$SST = n_1 ((A_1 - T)^2 ) + n_2 ((A_2 - T)^2 )$

## 4.) Calculate the Error Sum of Squares (SSE):

$$SSE = \sum_{1}^{n_1} (\sigma_1 - A_1)^2 + \sum_{2}^{n_2} (\sigma_2 - A_2)^2$$

## 5.) Calculate the Treatments Mean Square (MST) & Error MeanSquare (MSE):

$MST = SST / 1$ $\quad\quad$ $MSE = SSE /( (n_1 - 1) + (n_2 - 1) )$

## 6.) Calculate the F-Statistic (F):

$F = MST / MSE$

## 7.) Determine the Critical F-Statistic (F critical):

Look this value up in a F Distribution Table using 1% probability values

Numerator Degrees of Freedom = 1

Denominator Degrees of Freedom = (n1 -1) + (n 2 - 1)

## 8.) Compare F-Statistics:

If F < F critical then the stockpiles are the same

If F > F critical then the stockpiles are not the same

## Critical Values for F Distribution

### (1% Probability Values & 1 Degree of Freedom)

| **Degrees of Freedom for Denominator** | **F critical** |
|---|---|
| 1 | 40.52 |
| 2 | 98.49 |
| 3 | 34.12 |
| 4 | 21.20 |
| 5 | 16.26 |
| 6 | 13.74 |
| 7 | 12.25 |
| 8 | 11.26 |
| 9 | 10.56 |
| 10 | 10.04 |
| 11 | 9.65 |
| 12 | 9.33 |
| 13 | 9.07 |
| 14 | 8.86 |
| 15 | 8.68 |
| 16 | 8.53 |
| 17 | 8.40 |
| 18 | 8.28 |
| 19 | 8.18 |
| 20 | 8.10 |
| 21 | 8.02 |
| 22 | 7.94 |
| 23 | 7.88 |
| 24 | 7.82 |
| 25 | 7.77 |
| 26 | 7.72 |
| 27 | 7.68 |
| 28 | 7.64 |
| 29 | 7.60 |
| 30 | 7.56 |
| 32 | 7.50 |
| 34 | 7.44 |

| | |
|---|---|
| 36 | 7.39 |
| 38 | 7.35 |
| 40 | 7.31 |
| 42 | 7.27 |
| 44 | 7.24 |
| 46 | 7.21 |
| 48 | 7.19 |
| 50 | 7.17 |
| 55 | 7.12 |
| 60 | 7.08 |
| 65 | 7.04 |
| 70 | 7.01 |
| 80 | 6.95 |
| 100 | 6.90 |

## Example: Pooled t-Test Calculation

1. $A_1$ = (60+58+52+59+56+64+65+51+61+57+59+62+60+64+63) /15 = **59.33**

   $A_2$ = (54+63+58+51+49) /5 = **55.00**

2. T = (60+58+52+59+56+63+65+51+61+57+59+62+60+64+63+54+63+58+51+49) / (15+5) = **58.25**

3. SST = 5 ((55.00 - 58.25)$^2$ ) + 15 ((59.33 - 58.25)$^2$ ) = **70.31**

4. SSE = 233.35 + 126 = **359.35**

| | |
|---|---|
| 60 - 59.33 = 0.67 | $(0.67)^2 = 0.45$ |
| 58 - 59.33 = -1.33 | $(-1.33)^2 = 1.77$ |
| 52 - 59.33 = -7.33 | $(-7.33)^2 = 53.73$ |
| 59 - 59.33 = -0.33 | $(-0.33)^2 = 0.11$ |
| 56 - 59.33 = -3.33 | $(-3.33)^2 = 11.09$ |
| 63 - 59.33 = 3.67 | $(3.67)^2 = 13.47$ |
| 65 - 59.33 = 5.67 | $(5.67)^2 = 32.15$ |
| 51 - 59.33 = -8.33 | $(-8.33)^2 = 69.39$ |
| 61 - 59.33 = 1.67 | $(1.67)^2 = 2.79$ |
| 57 - 59.33 = -2.33 | $(-2.33)^2 = 5.43$ |
| 59 - 59.33 = -0.33 | $(-0.33)^2 = 0.11$ |
| 62 - 59.33 = 2.67 | $(2.67)^2 = 7.13$ |
| 60 - 59.33 = 0.67 | $(0.67)^2 = 0.45$ |
| 64 - 59.33 = 4.67 | $(4.67)^2 = 21.81$ |
| 63 - 59.33 = 3.67 | $\underline{(3.67)^2 = 13.47}$ |
| | Total = **233.35** |

| | |
|---|---|
| 54 - 55 = -1 | $(-1)^2 = 1$ |
| 63 - 55 = 8 | $(8)^2 = 64$ |
| 58 - 55 = 3 | $(3)^2 =9$ |
| 51 - 55 = -4 | $(-4)^2 = 16$ |
| 49 - 55 = -6 | $\underline{(-6)^2 = 36}$ |
| | **Total = 126** |

5. MST = 70.31/ 1 = **70.31**

   MSE = 359.35 / ( (15 - 1) + (5 - 1) ) = **19.96**

6. F Statistic = 70.31 / 19.96 = **3.52**

7. F Critical = **8.28** (DF = 18 for denominator)

8. F Statistic = 3.52 < F Critical = 8.28

   Stockpiles are the same.